wjec cbac

GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
FURTHER MATHEMATICS
UNIT 4 FURTHER PURE MATHEMATICS B
1305U40-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 4 FURTHER PURE MATHEMATICS B

SUMMER 2024 MARK SCHEME

Qu	Solution	Mark	Notes
1. a)	$r = \sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}$	B1	
	$\arg(5 + 10\text{i}) = 1.11$	B1	
	$\sqrt[3]{5 + 10\text{i}} = \sqrt[3]{\sqrt{125}\text{e}^{\frac{1.11\text{i}}{3}\left(+\frac{2n\pi\text{i}}{3}\right)}}$	M1	
	$z_1 = \sqrt{5}\text{e}^{0.37\text{i}}$	A1	One root
	$z_2 = \sqrt{5}\text{e}^{2.46\text{i}}$	m1	$+\frac{2n\pi\text{i}}{3}$
	$z_3 = \sqrt{5}\text{e}^{4.56\text{i}}$	A1	All roots Use of degrees gains B1, B1, M1, A0, M1, A0 at most
		[6]	Penalise -1 for use of $r(\cos\theta + \text{i}\sin\theta)$ or $x + \text{i}y$, unless written as $r\text{e}^{\text{i}\theta}$
b)	METHOD 1: Converting to coordinates:		FT (a)
	$z_1 = (2.086, 0.807)$ $z_2 = (-1.741, 1.404)$ $z_3 = (-0.344, -2.209)$	B1	Any two, si
	Distance between $z_1 z_2$: $\sqrt{(-1.741 - 2.086)^2 + (1.404 - 0.807)^2}$	M1	Or $z_1 z_3$, $z_2 z_3$
	$= 3.87(3285 \ldots)$	A1	Exact distance is $\sqrt{15}$
	Area = $\frac{1}{2} \times 3.87 \ldots \times 3.87 \ldots \times \sin\frac{\pi}{3}$	M1	Use of $\frac{1}{2}ab\sin C$, including degrees
	$= 6.5$	A1	cao Must be to 2 sf
	METHOD 2: Splitting triangle into 3 smaller isosceles triangles Angle at centre where 3 isosceles triangles meet = $\frac{2\pi}{3}$	(B1)	si
	Area of 1 isosceles triangle = $\frac{1}{2} \times 125^{\frac{1}{6}} \times 125^{\frac{1}{6}} \times \sin\frac{2\pi}{3}$	(M1) (A1)	Fully correct
	Area of full triangle $= 3 \times \frac{1}{2} \times 125^{\frac{1}{6}} \times 125^{\frac{1}{6}} \times \sin\frac{2\pi}{3}$	(B1)	Area × 3
	$= 6.5$	(A1)	cao Must be to 2 sf
		[5]	
		Total [11]	

Qu	Solution	Mark	Notes
2. a)	METHOD 1: Use of Maclaurin series, $\cosh x$ with $\left(\frac{x}{2}\right)$		
	$\cosh\left(\frac{x}{2}\right) = 1 + \frac{\left(\frac{x}{2}\right)^2}{2!} + \frac{\left(\frac{x}{2}\right)^4}{4!}$	M1	
	$\cosh\left(\frac{x}{2}\right) = 1 + \frac{x^2}{8} + \frac{x^4}{384}$	A1	
	METHOD 2: Use of Maclaurin series $f(x)$ with $\left(\frac{x}{2}\right)$		
	$f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0) + \frac{x^4}{24}f''''(0)$		
	$f(x) = 1 + (x \times 0) + \left(\frac{x^2}{2} \times \frac{1}{4}\right) + \left(\frac{x^3}{6} \times 0\right) + \left(\frac{x^4}{24} \times \frac{1}{16}\right)$	(M1)	
	$f(x) = 1 + \frac{x^2}{8} + \frac{x^4}{384}$	(A1)	
		[2]	
b)	$1 + \frac{x^2}{8} + \frac{x^4}{384} = x^2 - 2$	M1	FT (a)
	$\frac{x^4}{384} - \frac{7}{8}x^2 + 3 = 0$ $x^4 - 336x^2 + 1152 = 0$	A1	
	Solving e.g. if $y = x^2$, $y^2 - 336y + 1152 = 0$	m1	Solving quartic m0 no working
	$y = 332.5 \ldots$ or $y = 3.46 \ldots$	A1	cao
	Therefore, $x = (\pm)18.2 \ldots$ or $x = (\pm)1.86 \ldots$ Since $0 < x < 4$, at the point *A*, $x = 1.86 \ldots$	B1	cao Unsupported answer 0 marks
		[5]	
c)	Intersection of $g(x)$ and x-axis at $x = \sqrt{2}$	B1	si
	METHOD 1 (Integration of cosh): Area $= \int_0^{1.86} \cosh\left(\frac{x}{2}\right) dx - \int_{\sqrt{2}}^{1.86} (x^2 - 2) dx$	M1	FT (b), limits not required
	$= \left[2\sinh\left(\frac{x}{2}\right)\right]_0^{1.86} - \left[\frac{x^3}{3} - 2x\right]_{\sqrt{2}}^{1.86}$	A2	A1 each integration
	$= 2.139 \ldots \ldots - 0.310 \ldots \ldots$	m1	correct limits used
	$= 1.829$	A1	cao
	METHOD 2 (Use of Maclaurin expansion): Area $= \int_0^{1.86} \left(1 + \frac{x^2}{8} + \frac{x^4}{384}\right) dx - \int_{\sqrt{2}}^{1.86} (x^2 - 2) dx$	(M1)	FT (a) & (b), limits not required
	$= \left[x + \frac{x^3}{8 \times 3} + \frac{x^5}{384 \times 5}\right]_0^{1.86} - \left[\frac{x^3}{3} - 2x\right]_{\sqrt{2}}^{1.86}$	(A2)	A1 each integration
	$= 2.139 \ldots \ldots - 0.310 \ldots \ldots$	(m1)	correct limits used
	$= 1.829$	(A1)	cao
		[6]	
		Total [13]	

Qu	Solution	Mark	Notes
3.	Dividing both sides by $\cos x$:	M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y \sin x}{\cos x} = 4\cos^2 x \sin x + \frac{5}{\cos x}$		
	Integrating factor: $\mathrm{e}^{\int \frac{\sin x}{\cos x} \mathrm{d}x}$	M1	oe
	$= \mathrm{e}^{\int \tan x \mathrm{d}x} = \mathrm{e}^{\ln \sec x} = \sec x$	A1	
	THEN, METHOD 1:		
	Multiplying both sides:		
	$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \frac{\sin x}{\cos x} \sec x = 4 \sin x \cos x + \frac{5}{\cos x} \sec x$	M1	
	$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \tan x \sec x = 2 \sin 2x + 5 \sec^2 x$	A1	Form to integrate
	Integrating	m1	Both sides of equation
	$y \sec x = -\cos 2x + 5 \tan x \ (+c)$	A1	
	Substituting $y = 3\sqrt{2}$ when $x = \frac{\pi}{4}$:	m1	
	$3\sqrt{2} \times \sqrt{2} = 0 + 5 \times 1 + c \rightarrow \quad c = 1$		
	Solution: $y \sec x = -\cos 2x + 5 \tan x + 1$		
	$y = -\cos 2x \cos x + 5 \sin x + \cos x$	A1	cao Must be $y = \cdots$
	OR, METHOD 2:		
	Multiplying both sides:		
	$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \frac{\sin x}{\cos x} \sec x = 4 \sin x \cos x + \frac{5}{\cos x} \sec x$	(M1)	Form to integrate
	$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \tan x \sec x = 4 \sin x \cos x + 5 \sec^2 x$	(A1)	
	Integrating:	(m1)	Both sides of equation
	$y \sec x = 2 \sin^2 x + 5 \tan x \ (+c)$	(A1)	
	Substituting $y = 3\sqrt{2}$ when $x = \frac{\pi}{4}$:	(m1)	
	$3\sqrt{2} \times \sqrt{2} = 2 \times \frac{1}{2} + 5 \times 1 + c \rightarrow \quad c = 0$		
	Solution: $y \sec x = 2 \sin^2 x + 5 \tan x$		
	$y = 2 \sin^2 x \cos x + 5 \sin x$	(A1)	cao Must be $y = \cdots$

Qu	Solution	Mark	Notes
3.	OR, METHOD 3: Multiplying both sides: $\sec x\frac{\mathrm{d}y}{\mathrm{d}x}+y\frac{\sin x}{\cos x}\sec x=4\sin x\cos x+\frac{5}{\cos x}\sec x$ $\sec x\frac{\mathrm{d}y}{\mathrm{d}x}+y\tan x\sec x=4\sin x\cos x+5\sec^2 x$	(M1) (A1)	Form to integrate
	Integrating: $y\sec x=-2\cos^2 x+5\tan x\ (+c)$	(m1) (A1)	Both sides of equations
	Substituting $y=3\sqrt{2}$ when $x=\frac{\pi}{4}$: $3\sqrt{2}\times\sqrt{2}=-2\times\frac{1}{2}+5\times 1+c\rightarrow\quad c=2$	(m1)	
	Solution: $y\sec x=-2\cos^2 x+5\tan x+2$ $y=-2\cos^3 x+5\sin x+2\cos x$	(A1)	cao Must be $y=\cdots$
		[9] **Total [9]**	

Qu	Solution	Mark	Notes
4. a)	$\left(z+\frac{1}{z}\right)^4 = z^4+4z^2+6+4z^{-2}+z^{-4}$	M1 A1	3 correct terms unsimplified
	$=(z^4+z^{-4})+(4z^2+4z^{-2})+6$	m1	si
	$=2\cos 4\theta+8\cos 2\theta+6$	A1	
	$(2\cos\theta)^4=2\cos 4\theta+8\cos 2\theta+6$ $\therefore 16\cos^4\theta=2\cos 4\theta+8\cos 2\theta+6$	A1	cao
		[5]	
b)	$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(3-4\cos^2\theta)^2\,\mathrm{d}\theta$	M1	Condone omission of ½ until final A1
	$=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(9-24\cos^2\theta+16\cos^4\theta)\,\mathrm{d}\theta$	A1	
	$24\cos^2\theta=24\times\frac{1}{2}(1+\cos 2\theta)$	B1	si
	$=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(9-12(1+\cos 2\theta)+2\cos 4\theta+8\cos 2\theta+6)\,\mathrm{d}\theta$	A1	FT for A1A1A1 (a) if in form $a\cos 4\theta+b\cos 2\theta+c$
	$=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(3-4\cos 2\theta+2\cos 4\theta)\,\mathrm{d}\theta$		
	$=\frac{1}{2}\left[3\theta-2\sin 2\theta+\frac{2}{4}\sin 4\theta\right]_{\pi/6}^{5\pi/6}$	A2	A1 for 2 terms correct
	$=\frac{1}{2}\left(\frac{15\pi}{6}+\frac{2\sqrt{3}}{2}-\frac{\sqrt{3}}{4}\right)-\frac{1}{2}\left(\frac{3\pi}{6}-\frac{2\sqrt{3}}{2}+\frac{\sqrt{3}}{4}\right)$	m1	Use of limits
	$=\pi+\frac{3\sqrt{3}}{4}$ or 4.44…..	A1	cao
		[8]	

Qu	Solution	Mark	Notes
4. c)	$x = r\cos\theta = 3\cos\theta - 4\cos^3\theta$	M1	
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3\sin\theta + 12\cos^2\theta\sin\theta$	A2	-1 each error
	When perpendicular to initial line:		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3\sin\theta + 12\cos^2\theta\sin\theta = 0$	m1	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0$
	$-3\sin\theta\,(1 - 4\cos^2\theta) = 0$		
	Therefore,		
	$-3\sin\theta = 0$ or $1 - 4\cos^2\theta = 0$		
	$\cos^2\theta = \frac{1}{4}$	m1	
	$\sin\theta = 0$ $\cos\theta = \pm\frac{1}{2}$	A1	
	When $\sin\theta = 0$ the curve does not exist.		
	When $\cos\theta = \pm\frac{1}{2}$,		
	$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$	A1	
	$r = 2$	A1	
	$\left(2, \frac{\pi}{3}\right), \left(2, \frac{2\pi}{3}\right)$		
		[8]	
		Total [21]	

Qu	Solution	Mark	Notes
5. a)	$\frac{3-x}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$	M1	Use of
	$3 - x = A(x^2+1) + x(Bx+C)$	A1	
	When $x = 0, 3 = A$	A1	
	Substituting values e.g. When $x = 1, 2 = 2A + B + C$ $\quad\therefore B + C = -4$ When $x = -1, 4 = 2A + B - C$ $\quad\therefore B - C = -2$	m1	Or comparing coefficients
	Solving, $B = -3$ and $C = -1$	A1	
	$\int \frac{3-x}{x(x^2+1)}\,\mathrm{d}x = \int \left(\frac{3}{x} + \frac{-3x-1}{x^2+1}\right)\mathrm{d}x$		
	$= \int \left(\frac{3}{x} - \frac{3x}{x^2+1} - \frac{1}{x^2+1}\right)\mathrm{d}x$	M1	FT A, B, C if ≠ 0
	$= 3\ln\lvert x\rvert - \frac{3}{2}\ln\lvert x^2+1\rvert - \tan^{-1}x + c$	A2	A1 for two parts
		[8]	
b)	METHOD 1:		
	$\frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}} = \frac{2\sinh x\cosh x}{\cosh x\sqrt{\cosh^2 x - 9}}$	M1	Rewrite $\sinh 2x$ and take out $\sqrt{\cosh^2 x}$
	$= \frac{2\sinh x}{\sqrt{\cosh^2 x - 9}}$	A1	
	Let $u = \cosh x$ $\frac{\mathrm{d}u}{\mathrm{d}x} = \sinh x$	M1	Or equivalent appropriate sub.
	$\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}}\,\mathrm{d}x = \int \frac{2\sinh x}{\sqrt{\cosh^2 x - 9}}\,\mathrm{d}x$		
	$= \int \frac{2}{\sqrt{u^2-9}}\,\mathrm{d}u$	A1	Form to integrate
	$= 2\cosh^{-1}\left(\frac{u}{3}\right) + c$ OR $2\left(\ln\left\lvert u + \sqrt{u^2-9}\right\rvert\right) + c$	A1	
	$= 2\cosh^{-1}\left(\frac{\cosh x}{3}\right) + c$ OR $2\left(\ln\left\lvert \cosh x + \sqrt{\cosh^2 x - 9}\right\rvert\right) + c$	A1	Mark final answer
		[6]	

Qu	Solution	Mark	Notes
5. b)	METHOD 2:		
	Let $u = \cosh^2 x$	(M1)	
	$\frac{\mathrm{d}u}{\mathrm{d}x} = 2\cosh x \sinh x = \sinh 2x$		
	$\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}}\,\mathrm{d}x = \int \frac{1}{\sqrt{u^2 - 9u}}\,\mathrm{d}u$	(A1)	
	$= \int \frac{1}{\sqrt{\left(u - \frac{9}{2}\right)^2 - \frac{81}{4}}}\,\mathrm{d}u$	(M1) (A1)	Attempt to complete the square
	$= \cosh^{-1}\left(\frac{u - \frac{9}{2}}{\frac{9}{2}}\right) + c$ OR $\ln\left\|\left(u - \frac{9}{2}\right) + \sqrt{\left(u - \frac{9}{2}\right)^2 - \frac{81}{4}}\right\| + c$	(A1)	
	$= \cosh^{-1}\left(\frac{\cosh^2 x - \frac{9}{2}}{\frac{9}{2}}\right) + c$ OR $\ln\left\|\left(\cosh^2 x - \frac{9}{2}\right) + \sqrt{\left(\cosh^2 x - \frac{9}{2}\right)^2 - \frac{81}{4}}\right\| + c$	(A1)	Mark final answer
		[6]	
		Total [14]	
	Across parts (a) and (b) – penalise -1 once only for no constant term		

Qu	Solution	Mark	Notes
6. a)	The 3 **planes** will intersect at a single **point**.	B2 [2]	B1 Because det $\mathbf{M} \neq 0$, there is a unique solution.
b)	METHOD 1 (Use of inverse matrix): If $\mathbf{MX} = \mathbf{N}$, then $\mathbf{X} = \mathbf{M}^{-1}\mathbf{N}$. Cofactor matrix: $\begin{pmatrix} -600 & 92 & 425 \\ -80 & 40 & -30 \\ 400 & -96 & -240 \end{pmatrix}$	B1	May be implied by later work At least 5 correct entries
	Inverse matrix $\mathbf{M}^{-1} = \frac{1}{-1040}\begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix}$	M1 A1	
	Therefore, $\mathbf{X} = \mathbf{M}^{-1}\mathbf{N}$ $= \frac{1}{-1040}\begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix}\begin{pmatrix} 2668 \\ 3402 \\ 4581 \end{pmatrix}$ $\mathbf{X} = \begin{pmatrix} 39 \\ 56 \\ 65 \end{pmatrix}$	m1	
	Single £39, Double £56, Family £65	A2	A1 for two solutions Unsupported answer 0 marks
	METHOD 2 (Simultaneous equations/row operations): Let £x, £y, £z be the price of single, double and family rooms, respectively $12x + 30y + 8z = 2668$ $18x + 25y + 20z = 3402$ $19x + 50y + 16z = 4581$	(B1)	Forming 3 correct equations
	Multiplying each equation to get same coefficient of 1 variable e.g. $120x + 300y + 80z = 26680$ $72x + 100y + 80z = 13608$ $95x + 250y + 80z = 22905$		
	Eliminating 1 variable to arrive at 2 simultaneous equations e.g. $48x + 200y = 13072$ $25x + 50y = 3775$	(M1) (A1)	May be for y or z variables
	Solving pair of simultaneous equations $x = 39, y = 56, z = 65$	(m1)	
	Single £39, Double £56, Family £65	(A2)	A1 for two solutions Unsupported answer 0 marks

Qu	Solution	Mark	Notes
6. b)	METHOD 3 (Row reduction to echelon form): Let £x, £y, £z be the price of single, double and family rooms, respectively $12x + 30y + 8z = 2668$ R1 $18x + 25y + 20z = 3402$ R2 $19x + 50y + 16z = 4581$ R3	(B1)	Forming 3 correct equations
	Row operations on one row to eliminate variable e.g. $2 \times \text{R2} - 3 \times \text{R1}$ to give $12x + 30y + 8z = 2668$ $-40y + 16z = -1200$ $19x + 50y + 16z = 4581$	(M1)	May be on any row
	Row operations on second row to eliminate two variables	(m1)	May be on any row
	Correct echelon form e.g. $12x + 30y + 8z = 2668$ $-40y + 16z = -1200$ $208z = 13520$	(A1)	Must have 0 in 1 row and 0 0 in another row
	Value of 1 variable correct i.e. one of $x = 39, y = 56, z = 65$	(A1)	
	Substituting into remaining rows Remaining two values $x = 39, y = 56, z = 65$		
	Single £39, Double £56, Family £65	(A1)	All correct Unsupported answer 0 marks
		[6] **Total [8]**	

Qu	Solution	Mark	Notes
7. a) i)	$16-6x-x^2=25-(x+3)^2$	B1	
	Mean value = $\frac{1}{1--3}\int_{-3}^{1}\frac{1}{\sqrt{16-6x-x^2}}\mathrm{d}x$	M1	
	$=\frac{1}{1--3}\int_{-3}^{1}\frac{1}{\sqrt{25-(x+3)^2}}\mathrm{d}x$		
	$=\frac{1}{4}\left[\sin^{-1}\left(\frac{x+3}{5}\right)\right]_{-3}^{1}$	B1	For $\sin^{-1}\left(\frac{x+3}{5}\right)$
	$=\frac{1}{4}\left(\sin^{-1}\frac{4}{5}\right)-\frac{1}{4}(\sin^{-1}0)$		
	$=0.232$	A1	If M0, award SC1 for 0.232 unsupported
		[4]	
ii)	Volume = $\pi\int_{-3}^{1}\left(\frac{1}{\sqrt{16-6x-x^2}}\right)^2\mathrm{d}x$	M1	FT completed square from (i) if possible Award B1 from (i) here if not awarded previously
	$=\pi\int_{-3}^{1}\frac{1}{25-(x+3)^2}\mathrm{d}x$	A1	
	$=\pi\left[\frac{1}{2\times5}\ln\left\|\frac{5+(x+3)}{5-(x+3)}\right\|\right]_{-3}^{1}$ OR $=\pi\left[\frac{1}{5}\tanh^{-1}\frac{x+3}{5}\right]_{-3}^{1}$	A1	
	$=\pi\left[\frac{1}{10}\ln\left\|\frac{9}{1}\right\|-\frac{1}{10}\ln\left\|\frac{5}{5}\right\|\right]$ OR $=\pi\left[\frac{1}{5}\tanh^{-1}\frac{4}{5}-\frac{1}{5}\tanh^{-1}0\right]$	m1	Use of limits
	$=\frac{\pi}{10}\ln 9$ or $0.69(0278459\ldots)$	A1	If M0, award SC1 for 0.69 unsupported
		[5]	
b)	$\lim_{a\to\infty}\int_{1}^{a}\frac{-8\mathrm{e}^{-2x}}{4\mathrm{e}^{-2x}-5}\mathrm{d}x$	M1	
	$=\lim_{a\to\infty}[\ln\|4\mathrm{e}^{-2x}-5\|]_{1}^{a}$	A1	
	$=\lim_{a\to\infty}[\ln\|4\mathrm{e}^{-2a}-5\|-\ln\|4\mathrm{e}^{-2}-5\|]$		
	$=[\ln\|-5\|-\ln\|4\mathrm{e}^{-2}-5\|]$		
	$=0.115$	A1	No workings, 0 marks
		[3]	
		Total [12]	

Qu	Solution	Mark	Notes
8. a)	METHOD 1:		
	$4x + 3 = \sinh y$		
	$4\dfrac{dx}{dy} = \cosh y$	M1	oe
	$4\dfrac{dx}{dy} = \pm\sqrt{1 + \sinh^2 y}$	m1	Use of identity
	$4\dfrac{dx}{dy} = \pm\sqrt{1 + (4x+3)^2}$	m1	Sub for $\sinh y$
	$4\dfrac{dx}{dy} = \pm\sqrt{16x^2 + 24x + 10}$	A1	oe
	$\dfrac{dx}{dy} = \dfrac{\pm\sqrt{16x^2 + 24x + 10}}{4}$		
	$\frac{dy}{dx} = \frac{4}{\sqrt{16x^2 + 24x + 10}}$ AND Justification e.g. graph of $\sinh^{-1} x$ e.g. derivative of $\sinh x$ is $\cosh x$ which is always positive.	A1	convincing, with explanation over the choice of +
	METHOD 2:		
	$\sinh y = 4x + 3$		
	$\cosh y \dfrac{dy}{dx} = 4$	(M1)	oe
	$\dfrac{dy}{dx} = \pm\dfrac{4}{\sqrt{1 + \sinh^2 y}}$	(m1)	Use of identity
	$\dfrac{dy}{dx} = \pm\dfrac{4}{\sqrt{1 + (4x+3)^2}}$	(m1)	Sub for $\sinh y$
	$\dfrac{dy}{dx} = \pm\dfrac{4}{\sqrt{16x^2 + 24x + 10}}$	(A1)	oe
	$\frac{dy}{dx} = \frac{4}{\sqrt{16x^2 + 24x + 10}}$ AND Justification e.g. graph of $\sinh^{-1} x$ e.g. derivative of $\sinh x$ is $\cosh x$ which is always positive.	(A1)	convincing, with explanation over the choice of +
		[5]	

Qu	Solution	Mark	Notes
8. b)	METHOD 1:		
	$y = \frac{\sinh 2x}{e^{-3x}} = \frac{e^{2x} - e^{-2x}}{2e^{-3x}}$	M1	Rewrite and substitute $\sinh 2x$
	$y = \frac{e^{5x}}{2} - \frac{e^{x}}{2}$	A1	
	$\frac{dy}{dx} = \frac{5e^{5x}}{2} - \frac{e^{x}}{2}$	A1	
	When stationary, $\frac{dy}{dx} = 0$	m1	
	$\frac{5e^{5x}}{2} - \frac{e^{x}}{2} = 0$		
	$\frac{e^{x}}{2}(5e^{4x} - 1) = 0$		
	$\frac{e^{x}}{2} = 0$ or $5e^{4x} - 1 = 0$	A1	
	$\frac{e^{x}}{2} = 0$ does not lead to a solution for x		
	$5e^{4x} - 1 = 0$		
	$x = \frac{1}{4}\ln\frac{1}{5}$		
	Therefore, there is only one stationary point.	A1	
	METHOD 2:		
	$e^{-3x}y = \sinh 2x$		
	$e^{-3x}\frac{dy}{dx} - 3e^{-3x}y = 2\cosh 2x$	(M1)	
	$e^{-3x}\frac{dy}{dx} - 3\sinh 2x = 2\cosh 2x$		
	$\frac{dy}{dx} = \frac{2\cosh 2x + 3\sinh 2x}{e^{-3x}}$	(A1)	
	$\frac{dy}{dx} = \frac{e^{2x} + e^{-2x} + \frac{3}{2}e^{2x} - \frac{3}{2}e^{-2x}}{e^{-3x}}$		
	$\frac{dy}{dx} = \frac{5}{2}e^{5x} - \frac{1}{2}e^{x}$	(A1)	
	When stationary, $\frac{dy}{dx} = 0$	(m1)	
	$\frac{5e^{5x}}{2} - \frac{e^{x}}{2} = 0$		
	$\frac{e^{x}}{2}(5e^{4x} - 1) = 0$		
	$\frac{e^{x}}{2} = 0$ or $5e^{4x} - 1 = 0$	(A1)	
	$\frac{e^{x}}{2} = 0$ does not lead to a solution for x		
	$5e^{4x} - 1 = 0$		
	$x = \frac{1}{4}\ln\frac{1}{5}$		
	Therefore, there is only one stationary point.	(A1)	

Qu	Solution	Mark	Notes
8. b)	METHOD 3: $e^{-3x}y = \sinh 2x$ $y = e^{3x}\sinh 2x$	(M1)	
	$\frac{dy}{dx} = 3e^{3x}\sinh 2x + 2e^{3x}\cosh 2x$	(A2)	A1 each part
	When stationary, $\frac{dy}{dx} = 0$ $e^{3x}(3\sinh 2x + 2\cosh 2x) = 0$	(m1)	
	$e^{3x} = 0$ or $3\sinh 2x + 2\cosh 2x = 0$	(A1)	
	$e^{3x} = 0$ does not lead to a solution for x $3\sinh 2x + 2\cosh 2x = 0$ $\frac{\sinh 2x}{\cosh 2x} = -\frac{2}{3} \rightarrow \tanh 2x = -\frac{2}{3}$ $2x = -0.8047 \ldots\ldots$ $x = -0.402 \ldots\ldots$ or $\frac{1}{4}\ln\frac{1}{5}$		
	Therefore, there is only one stationary point.	(A1)	
		[6]	
		Total [11]	

Qu	Solution	Mark	Notes
9.	Rewriting, $\sin 6\theta + \sin 2\theta = 3\cos 2\theta - 2\cos^2\theta + 1$		
	Use of summing formula, $2\sin\frac{6\theta+2\theta}{2}\cos\frac{6\theta-2\theta}{2} = 2\sin 4\theta\cos 2\theta$	M1 A1	
	Use of identity, $3\cos 2\theta - (2\cos^2\theta - 1) = 3\cos 2\theta - \cos 2\theta$ $= 2\cos 2\theta$	M1 A1	
	Therefore, $2\sin 4\theta\cos 2\theta = 2\cos 2\theta$ $2\sin 4\theta\cos 2\theta - 2\cos 2\theta = 0$	m1	FT provided M1M1, for method marks only
	$2\cos 2\theta\,(\sin 4\theta - 1) = 0$	A1	Solvable form
	$2\cos 2\theta = 0$ or $\sin 4\theta - 1 = 0$	m1	Either
	$\cos 2\theta = 0$ $2\theta = \frac{\pi}{2} + n\pi$ $\theta = \frac{\pi}{4} + \frac{n\pi}{2}$	A1	cao
	$\sin 4\theta = 1$ $4\theta = \frac{\pi}{2} + 2n\pi$ $\theta = \frac{\pi}{8} + \frac{n\pi}{2}$	A1	cao
		[9]	
		Total [9]	

Qu	Solution	Mark	Notes
10. a)	METHOD 1: Rearrange first equation: $\frac{dx}{dt} - 4x - 6e^{3t} = 2y$ $\frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t} = y$	M1	
	Substituting into second equation: $\frac{d}{dt}\left(\frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t}\right) = 6x + 8\left(\frac{1}{2}\frac{dx}{dt} - 2x - 3e^{3t}\right) + 15e^{3t}$	M1	
	$\frac{1}{2}\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 9e^{3t}$ $= 6x + 4\frac{dx}{dt} - 16x - 24e^{3t} + 15e^{3t}$	A2	-1 each error
	$\frac{1}{2}\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 10x = 0$	A1	Convincing
	$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 20x = 0$	[5]	
	METHOD 2: Differentiating: $\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 2\frac{dy}{dt} + 18e^{3t}$	(M1)	
	Substituting for $\frac{dy}{dt}$ $\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 2(6x + 8y + 15e^{3t}) + 18e^{3t}$ $\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 12x + 16y + 30e^{3t} + 18e^{3t}$	(A1)	
	Substituting for y $\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 12x + 8\left(\frac{dx}{dt} - 4x - 6e^{3t}\right) + 48e^{3t}$ $\frac{d^2x}{dt^2} = 4\frac{dx}{dt} + 12x + 8\frac{dx}{dt} - 32x - 48e^{3t} + 48e^{3t}$ $\frac{d^2x}{dt^2} = 12\frac{dx}{dt} - 20x$	(M1) (A1)	
	$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 20x = 0$	(A1) **(5)**	Convincing

Qu	Solution	Mark	Notes
10. b)	The auxiliary equation is: $m^2 - 12m + 20 = 0$ $(m-2)(m-10) = 0$ $m = 2$ or $m = 10$ Therefore, the general solution is: $x = A\mathrm{e}^{2t} + B\mathrm{e}^{10t}$	M1 A1	Condone use of x and y
	Differentiating $\frac{\mathrm{d}x}{\mathrm{d}t} = 2A\mathrm{e}^{2t} + 10B\mathrm{e}^{10t}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 4A\mathrm{e}^{2t} + 100B\mathrm{e}^{10t}$	M1 A1	Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$
	Substituting and solving $2A + 10B = 9$ $4A + 100B = 10$	m1	
	Solving, $A = 5$ and $B = -\frac{1}{10}$	A1	
	Therefore, $x = 5\mathrm{e}^{2t} - \frac{1}{10}\mathrm{e}^{10t}$	B1	cao Must be $x = f(t)$
		[7] **Total [12]**	